

Resistance distance in directed graphs

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Let $G = (V, E)$ be a strongly connected and balanced digraph with vertex set $V = \{1, \dots, n\}$. The Laplacian matrix of G is then the matrix (not necessarily symmetric) $L := D - A$, where A is the adjacency matrix of G and D is the diagonal matrix such that the row sums and the column sums of L are equal to zero. Let $L^\dagger = [l_{ij}^\dagger]$ be the Moore-Penrose inverse of L . We define the resistance between any two vertices i and j of G by $r_{ij} := l_{ii}^\dagger + l_{jj}^\dagger - 2l_{ij}^\dagger$. Some interesting properties of the resistance and the corresponding resistance matrix $[r_{ij}]$ will be discussed in the talk.

The classical distance d_{ij} between any two vertices i and j in G is the minimum length of all the directed paths joining i and j . Numerical examples show that the resistance distance between i and j is always less than or equal to the classical distance, i.e. $r_{ij} \leq d_{ij}$. However, no proof of this inequality is known. In the talk, we will show that this inequality holds for all directed cactus graphs. This is a joint work with Dr. R. Balaji and Professor R. B. Bapat.

Keywords: Laplacian matrix, Moore-Penrose inverse, Balanced directed graph, cofactor sums, Directed cactus graph